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ON THE NATURE OF INDUCTION[1]

ANY proposition is susceptible to two sorts of proof. We can adduce premises that directly imply it, or we can adduce premises that indirectly imply it because they imply the falsity of its contradictory alternatives. In inductive reasoning we prove universal propositions by adducing as premises the particular propositions furnished by experience. Formal logic tells us that the value of a particular proposition consists in its power to disprove its contradictory universal rather than to prove its subalternate universal. We might naturally suppose that the evidential function of experience as a knowledge of particulars, was to disprove universal statements rather than to prove them, and that if a universal conclusion was proved true by appeal to experience, the proof would be based upon the disproof or elimination of alternatives. That induction is actually and always of this indirect type of inference, and that as such it is properly expressed by a disjunctive syllogism in the negative mood (*modus tollendo ponens*), is what I wish to show.

There is, of course, no novelty in the conception of induction as a process of elimination. Mill's canons are efficacious because they embody implicitly the eliminative principle. In Hobhouse and Aikins, to mention only two of the modern logicians, the principle is explicitly recognized, and the chief problems of induction are treated, especially by Hobhouse, from that point of view. Yet so far as I am aware there has been nowhere an attempt to identify induction in all its phases with the kind of indirect inference known as the *reductio ad absurdum,* and it has seemed to me worth while to make that attempt for two reasons: First, because the several inductive methods when viewed from this standpoint appear not as a group of disconnected principles, but as an organic system and hierarchy which is applicable in its entirety to every inductive problem and in which each principle has its own function and virtue by which it supplements the defects of the principles that precede it; second, on account of the new light thrown by the indirect theory of induction upon the general epistemological problem of deriving universals from particulars.

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[1] Read at the annual meeting of the American Philosophical Association, at Cambridge, December, 1905.

And now, by way of introduction to the more positive treatment of the subject, let us consider some of the difficulties involved in what is still, I think, the usual conception of induction. Induction when treated as a mode of direct inference is divided into two kinds,—perfect and imperfect. In perfect induction, we reason that as these *A's* are *B's* and as these *A's* are all the *A's,* it must follow that all *A's* are *B's.* It is clear that what is called perfect induction is only possible when the total number of individuals making the class is limited. Thus we can prove by this method that all the months in the year have less than thirty-two days, or that all the flowers in the garden are fragrant, but not that all bodies gravitate or that all men are mortal. In these latter propositions, which are genuine universals, the classes contain an unlimited number of members, and experience can never supply us with more than an insignificant fraction of them. In imperfect induction, which is supposed to be a degenerate form of perfect induction, we boldly conclude that because an infinitesimal portion of a class has been observed to possess a certain property the whole class will have that property. The methods or canons of induction are the principles that inform us when we can and when we can not take the inductive leap.

Now there is one circumstance in particular which might lead us to suspect that there was something radically wrong with the notion that induction is a degenerate form of perfect induction. Neither the actual number of positive instances observed nor the ratio of that to the total number has anything whatever to do with the degree of validity possessed by the induction. Perfect induction is essentially quantitative, depending, as it does, upon observation of all the members of a given class. The canons that guide us in making the so-called imperfect induction are, on the other hand, essentially qualitative, and not, as we might suppose, imperfectly quantitative. That is to say, it is never a question of observing almost all, or a bare majority, or even an appreciable fraction of the whole number of material bodies, for example, as evidence for the inductive generalization that all bodies gravitate. We contrive in the few cases under our control to eliminate by the methods of difference and especially of agreement all the characteristics of bodies that could possibly cause their gravitation except those of extension and inertia, and on the strength of this elimination we unhesitatingly conclude that a material body, merely as such (and hence all material bodies), will gravitate.

And now that we have briefly considered the contradiction between inductive theory as exemplified in the supposedly archetypal syllogism of perfect induction, and inductive practise as exemplified in Mill's canons, we may look to see how this contradiction can be removed by treating induction as belonging essentially and exclusively to the indirect type of inference.

Every inductive problem indirectly, and the usual inductive problem directly, concerns the determination of a causal relation. A phenomenon occurs in which we are for some reason interested and we at once seek among its antecedents and consequents for phenomena which are related to it as cause and as effect. Defining a causal relation as the relation of universal concomitant presence, absence and variation of two phenomena, we must assume as the basal postulates of all induction (1) that every event has an antecedent and a consequent with which it is causally or universally related, and (2) that we can enumerate these possible causal relations, by the aid of perception and previous knowledge. Now let *M* be a phenomenon whose causal relations we are seeking to discover, and let *A* be an antecedent or consequent phenomenon which we suspect, or provisionally assume, to be, and which in reality is, causally related to *M;* we can then classify the possible causal relations of *M* with respect to *A* under five heads. This division may be briefly stated in the form of a disjunctive proposition which will constitute the major premise of a typical inductive syllogism. Thus we can say that

The cause or effect of *M* is either (1) a phenomenon symbolized by *X* that is related to *A* only casually or by chance; or (2) a phenomenon symbolized by *B, C* or *D,* which is collocated with *A* but not indissolubly; or (3) a complex phenomenon symbolized by *A B, A C* or *A D,* of which *A* is an indispensable part; or (4) a phenomenon symbolized by *a* which is an aspect, phase or degree of *A;* or (5) *A* itself. The four inductive methods of simple enumeration, difference, agreement and concomitant variation express the types of particular negative propositions furnished by experience, and as such they constitute the complex minor premises of the syllogism and serve to contradict or eliminate all but one of the alternatives set forth in the major premise. The conclusion is, of course, the categorical affirmation of the only alternative not eliminated. I shall now try to show how each of the inductive methods is especially suited to eliminate one of these alternatives and that the eliminative function is the only function that they could or do perform: First, then, to remove the possibility that *A* and *M* are connected merely by chance, we use the method of simple enumeration. We observe the frequency with which *M* occurs in conjunction with *A,* and compare this with the frequency with which *M* might be expected to occur with A if they were quite independent. If the former frequency greatly exceeds the latter we consider the conjunction to be something more than casual. The *number* of observations required to eliminate the hypothesis of chance is thus strictly determined by the joint independent probability of the events in question.

Having eliminated chance by the method of simple enumeration, we next eliminate by the method of difference the possibility that M is causally related, not to A, but to the antecedents and consequents with which A is collocated. We find, let us say, cases in which B, C and D are simultaneously or successively present when the event M is absent. The results of such observations may be stated in the form of a particular negative proposition, as follows: Some cases of B, C and D are not cases of M, which means that the universal affirmative proposition: All cases of B, C or D are cases of M—which expresses a possible causal relation—is eliminated.

Supposing, now, that the phenomenon A has not been found present in the absence of M, and consequently has not been eliminated, it becomes necessary to apply the method of agreement in order to decide whether M is not causally related to a complex phenomenon such as AB, AC or AD, of which A is only a part, for the weak point in the method of difference lies in the fact that it can only prove that A is at least a part of the cause or effect of M, not that it is the whole. We observe by the method of agreement that B, C and D can be simultaneously or successively absent when M and A are both present. This again eliminates B, C and D, but it also eliminates the possibility that A needs to cooperate with B, C and D in order to be causally related to M.

We have now proved that M is causally related either to A or to some phase of A which we called a. And to secure the elimination of this fourth alternative we use the last and most powerful of the inductive canons, *viz.*, that of concomitant variation. If we find that M and A vary in perfect concomitance we know that every phase or degree of A, rather than some particular phase such as a, is causally related to every phase or degree of M. If, on the other hand, we had discovered that A did not vary with M in any manner, we should have proved that the true cause or effect of M was a and not A, as such. Or, again, if we had found that M did not vary directly with A, but with some function of A, we should conclude that the cause or effect of M was A in conjunction with a.

The universal affirmative conclusion of an inductive syllogism is thus in any case the result of successive eliminations—in the form of the particular negative propositions furnished by experience—of all but one of the alternative universals set forth in the disjunctive major premise as hypotheses. And each of the inductive methods is, as we have seen, adapted to the elimination of a certain type of alternative.

And now a word must be said in regard to the two methods which we have not mentioned—the method of residues and the joint method of agreement and difference. The method of residues is confessedly a method of elimination; it is, however, hardly worthy of being ranked with the other methods, for it is applicable only when both the antecedents and consequents are quantities of matter or energy, for in no other case can we apply the conception of a cause exhausting its causality in the production of a given effect. As for the joint method, its continued existence in logical text-books affords a good illustration (1) of the fact that logicians have failed to recognize the exclusively eliminative nature of induction and (2) of the results of that failure. The joint method bids us supplement the method of agreement, by the collection of as many different instances as possible of the absence of a phenomenon along with the absence of its supposed cause. Now it can easily be shown that these cases of concurrent absence are as such quite worthless as evidence of causal connection. If we are considering whether a protective tariff causes national prosperity we do not adduce as evidence the generation of Roentgen rays or the constructing of a sonnet, and yet these are different cases of the concurrent absence of protection and prosperity and as such perfectly conform to the requirements of the joint method as worded by Mill and as symbolized by Jevons. What we actually seek to find in such an investigation are always cases in which not merely the supposed cause is absent, but in which *the alternatives to the supposed cause are present* along with the absence of the effect, and hence are eliminated. In the method of difference as usually schematized this is done in a single pair of instances in which ABC followed by M is compared with BC followed by the absence of M; but it can equally well be done piecemeal or by a succession of instances, one showing simply the presence of B, a second the presence of C, a third the presence of D, etc.; along with the absence of M. Now as the so-called joint method is, when rightly understood, nothing whatever but a combination of the method of agreement and the method of difference where each is applied successively in several instances, rather than simultaneously in a single pair, it does not deserve to be classed as a separate canon.[2]

The claim was made at the beginning of the paper that the identification of induction with the indirect type of argument, or *reductio ad absurdum,* possessed two advantages, (1) the unification of the inductive methods, (2) the exhibition in a new light of the general epistemological problem of deriving universals from particulars. I have said what I could in regard to the first of these advantages, but I should like in conclusion to speak further as to the second.

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[2] By way of illustration I subjoin (1) the incorrect and meaningless symbolization of the joint method given by Jevons, 'Lessons in Logic,' p. 247; (2) the correct symbolization substantially as given by Aikins, 'The Principles of Logic,' p. 275; (3) the symbolization in a single pair of instances of the

The attacks upon the possibility of reasoning from particulars, that have been made by the sceptics on the one hand and by the extreme apriorists on the other, are based in the main upon a quite proper realization of the gulf between a subaltern proposition and its subalternates. The number of cases exemplifying a genuine universal or law of nature is, as we have said, always infinite, and hence the direct inference from some to all is not only uncertain (which would be admitted by inductive logicians), but would seem to be not even probable—to be, in fact, infinitely improbable. For we can, of course, never observe even an appreciable fraction, to say nothing of a majority of the members of an infinite series. Now when we give up this attempt at direct inference and exorcise from inductive theory the specter of a so-called perfect induction as an ideal to be approximated, the whole problem appears in a less paradoxical and more hopeful light. For the experiential evidence in the form of particular propositions, which was worthless as a means of direct proof of the subalternate, is perfectly capable of disproving the contradictory and thus indirectly establishing a hypothesis as a survival of the fittest. Of course this does not mean that we have merely, by the substitution of the intensive for the extensive view of the subject, removed uncertainty from generalizations from experience, but only that from our point of view we may more clearly see why it is that the degree of probability of any inductive conclusion is measured by the number of antecedently possible alternative conclusions and by the ease with which they can be isolated, enumerated and eliminated rather than by the mere number of instances observed.

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method of agreement and of the method of difference. The substantial identity of (2) and (3) is obvious.

(1)	(2)	(3)
ABC — abc	*ABCD — abcd*	*ABC — abc*
ADE — ade	*ABGH — abgh*	*ADE — ade*
AFG — afg	*ACGK — acgk*	
AHK — ahk		
PQ — pq	*BDE — bde*	*ABC — abc*
RS — rs	*CDG — cdg*	*BC — bc*
TV — tv	*BEF — bef*	
XY — xy		